EFFICIENCY OF CONDUCTOR ACCELERATION IN THE PULSED MAGNETIC FIELD OF A SOLENOID

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UDC 538.323:531.551

One of the methods of obtaining supersonic velocities of solids to investigate phenomena related to high-velocity collisions is the induction acceleration of conductors in a pulsed magnetic field. The processes which occur in electromagnetic induction ring-conductor accelerators have been fairly fully investigated [1-3], and have found practical application. To investigate high-velocity interaction it is necessary to obtain high velocities with bodies of different shape, particularly cylindrical bodies. Such conductors can be accelerated in the pulsed magnetic field of an inductor having the shape of a solenoid [4].

In Fig. 1 we show a theoretical sketch of an induction solenoid accelerator. The system of differential equations describing the electromechanical transients when a capacitive energy store is discharged through the solenoid which has a cylindrical conductor arranged coaxially inside it has the following dimensionless form:

$$\rho_1 j_1 + (1+\lambda_0) dj_1/d\tau + \mu dj_2/d\tau + j_2 \nu d\mu/d\varepsilon = \varphi; \quad (1)$$

$$\rho_2 j_2 + \lambda dj_2/d\tau + \mu dj_1/d\tau + j_1 \nu d\mu/d\varepsilon = 0; \quad (2)$$

$$d\varphi/d\tau = -j_1; \quad (3)$$

$$d\varepsilon/d\tau = \nu; \quad (4)$$

$$\frac{d^2\varepsilon}{d\tau^2} = \frac{j_1 j_2}{\sigma}\frac{d\mu}{d\varepsilon}. \quad (5)$$

The initial conditions when $\tau = 0$: $j_1(0) = j_2(0) = 0$, $\varphi(0) = 1$, $\varepsilon(0) = \varepsilon_0$, $\nu(0) = \begin{cases} 0 \\ \nu_0. \end{cases}$ The dimensional and dimensionless quantities are related as follows:

$$j_{1,2} = i_{1,2}/i_b,\ \ \varphi = U/U_b,\ \ \tau = t/t_b,\ \ \varepsilon = x/x_b,\ \ \lambda_0 = L_0/L_b,\ \ \mu = M/M_b,$$
$$\lambda = L_2/L_b,\ \rho = R/R_b,\ \nu = v/v_b,\ \sigma = m/m_b.$$

The basis quantities are

$$x_b = d_1,\ \ U_b = U_0,\ \ L_b = M_b = L_1,\ \ i_b = U_0\sqrt{C/L_1},\ \ R_b = \sqrt{L_1/C},$$
$$t_b = \sqrt{L_1 C},\ \ v_b = d_1/\sqrt{L_1 C},\ \ m_b = C^2 U_0^2 L_1/d_1^2,$$

where C, U_0, R_0, L_0 are the capacitance, the initial voltage, the resistance, and inductance of the capacitive energy store; L_1, R_1, and d_1 are the inductance, resistance, and mean diameter of the solenoid; L_2, R_2, and m are the inductance, resistance, and mass of the accelerated conductor; M is the mutual inductance between the inductor and the conductor; and x is the distance between the geometrical centers of the inductor and the accelerated conductor.

The main assumption which is made in the theoretical model of the accelerator compared with an actual accelerator is that the inductor and the massive cylindrical conductor are replaced by thin solenoids in which the current density is uniformly distributed along their length, while the thickness of the current layer is equal to the equivalent depth of penetration of the electromagnetic field into the metal of the inductor and the accelerated body.

A comparison of the results of a calculation using this mathematical model with the results given in [4], obtained by the method of integral equations, and also with the results of experimental investigations, confirms the satisfactory agreement between the integral characteristics of the accelerator, particularly of the discharge current in the inductor and the final velocity of the accelerated conductor.

Istra. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 82-86, March-April, 1983. Original article submitted January 8, 1982.

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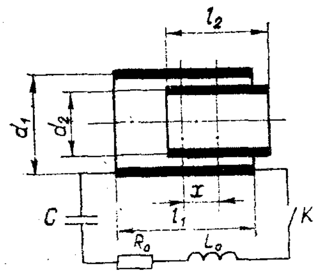

Fig. 1

The system of differential equations (1)-(5) were integrated on the ES-1020 computer using the Runge–Kutta method with variable step. The effect of heating of the inductor and the accelerated conductor on the energy conversion process was ignored. The mutual inductance between the inductor and the conductor was calculated from the equation [5]

$$M = \frac{\pi}{8}\mu_0 N_1 N_2 \frac{d_2^2}{l_1 l_2}(l_{01}F_1 - l_{02}F_2 - l_{03}F_3 + l_{04}F_4),$$

where N_1 and N_2 are the number of turns of the solenoids, l_{0i} is a geometrical quantity determined by the mutual position of the solenoids,

$$\mu_0 = 4\pi\cdot 10^{-7}\text{H/m};\ l_{0i} = \sqrt{(d_1/2)^2 + x_i^2};\ i = 1, 2, 3, 4;$$
$$x_1 = x + (l_1 + l_2)/2;\quad x_2 = x - (l_1 - l_2)/2;$$
$$x_3 = x + (l_1 - l_2)/2;\quad x_4 = x - (l_1 + l_2)/2;$$

and F_i is a tabulated quantity which was approximated, with an error of 2-6%, by the expression $F_i = 1 - 0.024(d_2/l_{0i})^2$.

The efficiency with which the energy of the capacitive store is converted into kinetic energy of the accelerated conductor can be estimated from the efficiency, which is given by the following equation in the general case when the conductor has initial velocity v_0:

$$\eta = [m(v^2 - v_0^2)]/(CU_0^2) = \sigma(v^2 - v_0^2).$$

Preliminary calculations showed that there is a slight maximum on the curve of the efficiency against the axial dimensions of the inductor $l_1^* = l_1/d_1$ and the conductor $l_2^* = l_2/d_1$. The initial position ($\varepsilon_0 = x_0/d_1$) of the geometrical center of the conductor with respect to the center of the inductor has a considerable effect on the efficiency. Henceforth, we will use the simplex method [6] to find the optimum values of l_1^*, l_2^*, and ε_0 for each fixed set of dimensionless parameters ρ_1, ρ_2, λ_0, σ, $d^* = d_2/d_1$. The initial regular simplex was constructed in factor space, formed by the three factors l_1^*, l_2^*, ε_0, the code units of which are chosen to be 0.1, 0.1, and 0.05. The optimum was assumed to be attained if one and the same point occurs five times in the sequential simplexes $P = 1.65n + 0.05n^2$, where n is the number

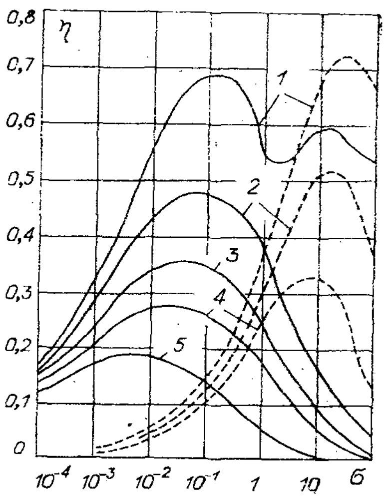

Fig. 2

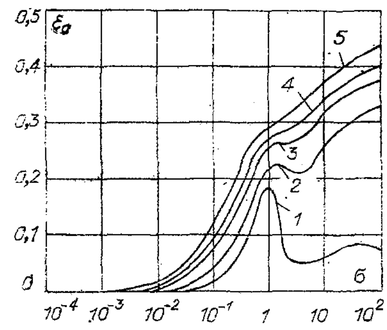

Fig. 3

of factors. We chose as the optimum parameters those which corresponded to the point with maximum acceleration efficiency.

In Fig. 2 we show curves of the efficiency of the solenoid accelerator (the continuous curves) as a function of the relative mass of the conductor for an initial velocity $\nu_0 = 0$, and optimum ε_0 and $\rho_1 = 0, 0.1, 0.2, 0.3$, and 0.5 (curves 1-5, respectively) for $\rho_2 = 0.5\rho_1$. Here and henceforth $\lambda_0 = 0.1$ and $d^* = 0.98$. It follows from the curves in Fig. 2 that the maximum energy conversion efficiency is obtained when $\sigma = 0.01$-0.1, when the acceleration occurs during the first half-period of the discharge current. As σ is increaed the efficiency falls when $\sigma > 0.1$ due to mismatch between the acceleration process and the increase in the fraction of the energy dissipated in resistive losses in the conductors. In the region where $\sigma < 0.01$, the efficiency decreases due to rapid departure of the conductor from the inductor–conductor interaction zone and the increase in the emf of the motion $e_m = i_2 vdM/dx$, without allowing the capacitive energy store to discharge completely during the acceleration period.

It is of interest to compare a solenoid accelerator and an accelerator with a plane inductor system, used to accelerate ring conductors [3], from the point of view of energy conversion efficiency. Curves of $\eta(\sigma)$ for a plane inductor system are shown in Fig. 2 (the broken curves). The comparison was made for the case when the mean diameters of the solenoid and the plane inductor were the same for a relative width of the current strip of the plane conductor $\alpha = 0.1$, where $\alpha = (r_1 - r_2)/(r_1 + r_2)$, and r_1 and r_2 are the external and internal radii of the inductor, respectively. It follows from the results of the calculations shown in Fig. 2 that for optimum relative masses of the accelerated conductors, the solenoid and plane inductor systems enable one to obtain approximately the same energy conversion efficiency. However, other conditions being equal, maximum acceleration efficiency in a solenoid inductor system is achieved for a relative conductor mass σ that is 100-200 times less than in the case of a plane inductor, which can be explained by the difference in the dependence of the mutual inductance on the conductor displacement.

When $\sigma > 1$, the efficiency of the plane inductor system is higher, since one can obtain higher values of the derivative $d\mu/d\varepsilon$, which is the decisive factor for small conductor displacements during the time the store discharges. When $\sigma < 1$ the efficiency of a plane inductor system is less than that of a solenoid system because of the rapid departure of the

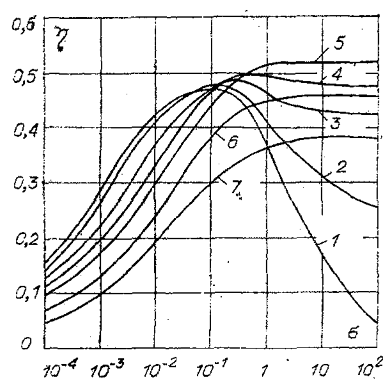

Fig. 4

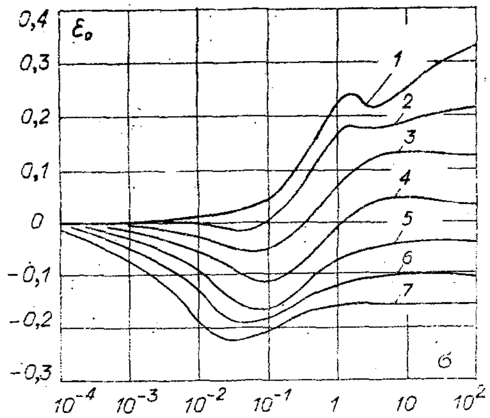

Fig. 5

conductor from the inductor—conductor interaction zone. In a plane inductor system the value of the inductor—conductor interaction zone is ε_k = 0.3-0.4, and in the solenoid system ε_k = 1.2-1.5. In addition, in a plane inductor system for small values of σ the efficiency falls due to limitation of the discharge current of the rapidly increasing emf of the motion, whereas in the solenoid-type inductor system one can obtain a small initial value of $d\mu/d\varepsilon$ by choosing the optimum value of ε_0, and during the period in which the current grows the emf of the motion is quite small. As a result, the acceleration occurs for higher values of the discharge current, which increases the efficiency of the solenoid system for small values of σ. The relative resistances of the discharge circuit have a greater effect on the efficiency of a solenoid system than on the efficiency of a plane system, so that for the same ρ_1 and optimum values of the relative mass σ the efficiency of a plane-inductor accelerator is higher than that of a solenoid system (see Fig. 2).

The initial position of the conductor in the inductor, determined by the distance between their geometrical centers ε_0, has an important effect on the efficiency of the solenoid accelerator. In the case of a conductor that is fixed at the initial instant of time, its optimum position in the inductor satisfies the condition

$$0 < \varepsilon_0 < \varepsilon_M,$$

where ε_M is the position of the center of the conductor in the inductor, for which the value of the derivative $d\mu/d\varepsilon$ is a maximum.

Curves of the optimum values of ε_0 as a function of σ and ρ_1 are shown in Fig. 3 (curves 1-5 correspond to ρ_1 = 0, 0.1, 0.2, 0.3, and 0.5). When the relative mass of the conductor increases, and when the accelerator characteristics deteriorate due to the increase in the resistive losses, the optimum initial position of the conductor shifts towards the exit of the inductor, approaching the coordinate ε_M but not reaching it. When there are no resistive losses ($\rho_1 = \rho_2 = 0$), for relative masses $\sigma > 0.1$ the acceleration efficiency depends very much on the instant when the conductor leaves the inductor—interaction zone, i.e., whether at this instant the current has a maximum or a zero value. As a result of this, local extrema are observed on the $\eta(\sigma)$ and $\varepsilon_0(\sigma)$ curves. As the resistive losses increase the effect of the second and subsequent half-periods of the discharge current decreases, and the $\eta(\sigma)$ and $\varepsilon_0(\sigma)$ curves become smoothed out.

A distinguishing feature of the solenoid inductor system compared with a plane system is the possibility of using it to construct a cascade induction accelerator. In this connection

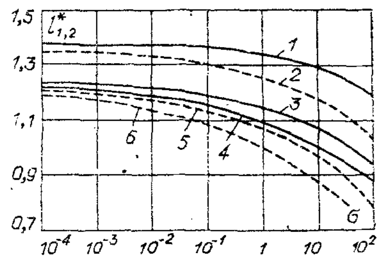

Fig. 6

we will consider the effect of the initial velocity of the conductor ν_0, corresponding to the instant when the energy store is connected to the inductor, on the energy conversion efficiency. In Fig. 4 we show the results of a calculation of the acceleration efficiency as a function of σ for $\rho_1 = 0.1$ and $\nu_0 = 0, 0.1, 0.3, 0.5, 0.7, 1.0$, and 1.5 (curves 1-7, respectively). The values of the relative initial distance ε_0 were chosen to be optimum. Maximum efficiency is obtained for an optimum initial velocity $\nu_0 = 0.5$-1.0. For an initial conductor velocity less than the optimum value, the acceleration efficiency falls due to the increase in the fraction of the energy dissipated in resistive losses, and for velocities greater than the optimum value the efficiency falls due to the rapid departure of the conductor from the conductor–inductor interaction zone.

The effect of the initial conductor velocity ν_0 on its optimum position, corresponding to the instant when the store is connected to the inductor, is illustrated by curves 1-7 in Fig. 5 ($\nu_0 = 0, 0.1, 0.3, 0.5, 0.7, 1$, and 1.5, respectively). When ν_0 is increased the optimum value of ε_0 shifts from the exit to the center of the inductor. When the initial velocities are increased further ($\nu_0 > 0.1$) it then becomes best to connect the energy store to the inductor before the conductor passes the center of the inductor ($\varepsilon_0 < 0$). In this case, some slowing down of the conductor is compensated by the fact that henceforth its acceleration occurs at higher values of the discharge current. The maxima on the $\eta(\sigma)$ curves correspond approximately to the cases when the connection of the store to the inductor at the instant when the conductor passes through the center of the inductor is optimum. The slight increase in efficiency when $\nu_0 = 0.5$-0.7 compared with $\nu_0 = 0$ is due to the fact that the optimum initial position of the conductor is shifted from the exit of the inductor to its center, thereby ensuring a greater multiplicity of the change in the overall inductance of the inductor system.

In Fig. 6 we show the effect of the initial velocity ν_0 on the optimum axial dimensions of the inductor and the conductor (the continuous and broken curves respectively; 1, 2: $\nu_0 = 0.7$, $\rho_1 = 0.1$; 3, 5: $\nu_0 = 0$, $\rho_1 = 0.1$; 4, 6: $\nu_0 = 0$, $\rho_1 = 0.3$). When $\nu_0 = 0$ the optimum length of the inductor l_1^* and the conductor l_2^* lie in the range 0.8-1.2, and $l_2^* < l_1^*$. An increase in the relative mass of the conductor and of the resistive losses in the discharge circuit leads to some reduction in the optimum values of l_1^* and l_2^*. When $\nu_0 \neq 0$ the optimum values of the axial dimensions increase, and when $\nu_0 = 0.5$-0.7 they are 1.1-1.4. This enables us to increase the time during which the conductor interacts with the magnetic field of the inductor and increases the energy conversion efficiency.

In conclusion we note the following.

1. For zero initial velocity the maximum acceleration efficiency of the conductor in the pulsed magnetic field of the solenoid is obtained for optimum values of the solenoid length $l_1^* = 0.8$-1.2, a conductor length $l_2^* = (0.8$-$0.9)l_1^*$, an initial displacement of the center of the conductor with respect to the center of the inductor $0 < \varepsilon_0 < \varepsilon_M$, and values of the relative mass $\sigma = 0.1$-0.01.

2. When the resistive losses in the discharge circuit increase the optimum values of the axial dimensions of the inductor and the conductor are reduced and the optimum initial displacement increases, approaching the value ε_M.

3. For an initial conductor velocity different from zero, the optimum initial position is displaced towards the entrance to the inductor, in which case, for the optimum initial velocity, it is best to connect the energy store to the inductor system when $\varepsilon_0 \approx 0$.

4. For the same values of the fundamental dimensionless parameters, a solenoid and plane inductor system under optimum conditions enable one to obtain similar values of the efficiency, but in a solenoid inductor system the optimum values of the relative mass of the conductor are 100-200 times less than for a plane inductor system.

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NUMERICAL INVESTIGATION OF THE PROCESSES IN A PLASMATRON WITH KEEN BLOWING

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High-enthalpy plasmatrons are an important element of new industrial technology at the present time because of the possibility of obtaining rapid heating of a wide range of gases (including chemically active gases such as hydrogen, oxygen, chlorine etc.) up to high temperatures ($2\text{-}3\cdot 10^4$°K) at working pressures of up to 10^7 Pa. One of the main problems in constructing such plasmatrons is to increase the specific energy contribution to the electric arc. Since the current flowing through the plasmatron has an upper limit set by the permissible erosion of the electrodes, to increase the power it is necessary to increase the electric field in the arc. One can obtain effective control of the electric field by blowing gas either in the intersection slots or through a porous interelectrode mounting [1]. One form of blowing is keen blowing, when gas is blown through one of the intersection slots with a flow rate of the order of or greater than the main flow. In this case the electric field in the arc and the temperature are increased, a gas curtain is formed on the walls of the plasmatron which enables the temperature of the wall to be maintained within the desired limits, and the boundary layer breaks up, leading to a hydrodynamically developed flow. Experimental investigations [1, 2] have been devoted mainly to analyzing the integral parameters: the current—voltage characteristics, the thermal efficiency, the mean-mass enthalpy, etc. This is mainly due to the difficulties involved in making measurements of the local characteristics, although when designing plasmatrons the latter play an important role.

To calculate the local parameters, numerical methods of analysis are promising, but even their use involves a number of difficulties. The complex structure of the flow of gas, and the impossibility of distinguishing the characteristic direction of motion make numerical calculations based on the boundary-layer approximation, which is mainly employed to investigate arcs in the plasmatron [3, 4], very inefficient. For a flow rate of the gas in the blowing through the walls of the order of the main flow rate, finite-difference algorithms cease to converge. Numerical simulation of the arc using the solution of the complete Navier—Stokes equations is the most acceptable. Using this method, investigations have been made [5] of the electric arc in the initial part of the plasmatron [6, 7] and in the case when there is intense blowing of gas through a porous wall [8]. These papers domonstrate the possibility of using numerical methods developed mainly for an incompressible liquid, and there is good qualitative agreement with experiments. In the present paper, using the complete Navier—Stokes equations we investigate the electric arc in a sectioned channel of a plasmatron with intense local gas blowing; the arc parameters are chosen to be such that a comparison can be made with experimental data [2] in order to estimate the efficiency of the method.

1. To describe the flow of an equilibrium plasma in the channel we will use the complete system of Navier—Stokes equations taking into account radiation transfer and Joule heat dissipation. By changing to the vortex-current function variable and using the assumptions employed in [3, 9], usually used when investigating an arc, the system of equations in the axisymmetrical stationary case can be reduced to the form [5]

$$r^2\left[\frac{\partial}{\partial z}\left(\frac{\Omega}{r}\frac{\partial\Phi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\frac{\Omega}{r}\frac{\partial\Phi}{\partial z}\right)\right]-\frac{\partial}{\partial z}\left[r^3\frac{\partial}{dz}\left(\mu\frac{\Omega}{r}\right)\right]- \quad (1.1)$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 87-91, March-April, 1983. Original article submitted January 26, 1982.

